Probability and Statistics
Course Code: MTH 103
Spring Semester - Final Exam.

Mansoura University
Faculty of Engineering

Biomedical Engineering Program
Level 100
Exam Date: 13-6- 2015
Allowed Time: 2 Hours

Attempt all questions. Assume any missed data. Full mark is 50

Q.1) The following scores represent the final examination grades for 20 students at the BME department.

75, 90, 95, 85, 80, 82, 60, 96, 85, 74, 85, 83, 62, 66, 77, 91, 69, 63, 92, 79

i.   Find the mean, median, mode, and coefficient of skewness (Use raw data)

ii.   Construct a frequency distribution table using 6 class intervals

iii. Sketch the histogram for this data

iv.   Find the average deviation, standard deviation , and coefficient of variation (Use grouped data).

v.   Determine P_{40} and D_5 using percentage ogive.   **[15 Marks]**

Q.2.a) The mean systolic blood pressure of a population of 15-45 years of age is 125 with a standard deviation or 12.5. If the blood pressure is normally distributed, what is the probability that a person of this group, selected at random, has a blood pressure below 140.   **[5 Marks]**

Q.2.b) Suppose two million lottery tickets are issued with 100 winning tickets among them. If a person purchases 100 tickets, what is the probability of winning? (b) How many tickets should one buy to be 95% confident of having a winning ticket?   **[5 Marks]**

Q.2.c) A random sample of size 20 taken from a normal distribution has a mean of 32.8 and a standard deviation of 4.51. Construct a 95% confidence interval of μ .   **[5 Marks]**

Q.3.a) A course in mathematics is taught to 12 students by the conventional classroom procedure. A second group of 10 students was given the same course by means of programmed materials. At the end of the semester the same examination was given to each group. The 12 students meeting in the classroom made an average grade of 85 with a standard deviation of 4, while the 10 students using programmed materials made an average of 81 with a standard deviation of 5. Test the hypothesis that the two methods of learning are equal using a 0.10 level of significance. Assume the populations to be approximately normal with equal variances.   **[5 Marks]**

Q.3.b) A manufacturer of car batteries claims that the life of his batteries has a standard deviation equal to 0.9 year. If a random sample of 10 of these batteries have a standard deviation of 1.2 years, do you think that σ > 0.9 year?   **[5 Marks]**

Q.3.c) It is desired to decide if Egyptians have a favorite team in the English Premier League or not. The following data has been obtained from a sample of 175 persons who follow English Premier League. Could you help to perform this test based on the following data?   **[5 Marks]**

Team	Tottenham	Aston Villa	Liverpool	Chelsea	Arsenal	Man. United	Man. City
No. of Fans	10	13	27	35	33	32	25

Q.4) The foreign exchange reserves (in Billion Dollars) in Egypt over 10-months period (After 25 January) is summarized in the following table.

Month	1	2	3	4	5	6	7	8	9	10
Reserves	36	34	33	30	28	26	25	23	21	20

i.   Plot the reserves versus month on a scatter diagram. Connect points to show results as a time series.

ii. Apply the moving average (order 3) to smooth data. Plot smoothed data on the scatter diagram using different color.

iii. Find the equation of the least square regression line that relates month to reserves. Plot the line on the scatter diagram.

iv. Could you guess when the foreign exchange reserves become zero?

v.   Calculate the correlation coefficient. Comment on result.   **[10 Marks]**

My best wishes to all of you!

Assis. Prof. Hossam El-Din Moustafa

Exam Guidelines: This Exam contains **5** questions in **2** pages. Start every question in a new page.

Question (1) [10 pt.]

a) [5 pt.] A box **I** contains 9 light bulbs of which 4 are defective, box **II** contains 9 light bulbs of which 5 are defective, and box **III** contains 9 light bulbs of which 3 are defective. We select a box at random and then select two bulbs at random from it. Find the probability that:

   i) the two bulbs are defective,

   ii) the two bulbs are selected from the urn **III** given that the two bulbs are defective.

b) [5 pt.] Prove that: if three events A, B and C are independent events, then $(A \cup B)$ and C are independent.

Question (2) [10 pt.]

a) [5 pt.] 10 students enter an exam of 50 *degree full mark*. Find the probability that:

   i) they got different degrees,

   ii) at least three students got the same degree.

b) [5 pt.] If the number of misprints, denoted by X, on a single page of a book has a Poisson distribution such that $P(X = 1) = P(X = 2)$.   **i)** What is the probability that there is at least two errors in this page,   **ii)** Find also, the probability that there is 5 errors in a given three pages.

Question (3) [10 pt.]

a) [5 pt.] When we request a call from base station, the call may be blocked with probability 0.1, terminated with probability 0.3, or make successfully with probability 0.6. If we request 10 calls, what is the probability to get:

i) 2 blocked and 5 success calls,

ii) 6 success calls.

iii) If call request continued until it make successfully, what is the probability that the first success call occurs at the 5$^{\text{th}}$ request.

b) [5 pt.] If the probability density function of a continuous random variable is given by

$$f(x) = \begin{cases} e^{2x} & x \leq 0 \\ kx & 0 < x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find: i) the value of k, (ii) the mean and variance, iii) $P(-0.5 \leq x \leq 0.5)$.

Question (4) [10 pt.]

a) [5 pt.] The time to failure of fans in a personal computer can be modeled by an exponential distribution with mean 4000 hours. Find the probability that the fans will fail:

(i) after 8000 hours, (ii) between 5000 and 10000 hours.

iii) Determine a such that the probability that the fans will fail before a hours is 0.75.

b) [5 pt.] The brightness of a television picture tube can be evaluated by measuring the amount of current required to achieve a particular brightness level. If it found that this follow normal distribution with standard deviation $\sigma = 15.7$. A sample of 25 tubes results in $\bar{x} = 317.2$ and $s = 15.5$. i) Find in a 95 % confidence interval for the mean of the current required. ii) Find the number of tubes, n, taken as a sample to reduce the confidence interval length to the halfe of it in part (i).

Question (5) [10 pt.]

a) [5 pt.] Consider the sugar content of the syrup in canned peaches. Suppose that the variance is thought to be $\sigma^2 = 18$ (milligrams)^2. A random sample of size $n = 9$ cans yields a sample standard deviation of $S = 4.8$ milligrams. Using 5 % level of significance; Test the hypothesis: $H_0: \sigma^2 = 18$ versus $H_1: \sigma^2 \neq 18$.

b) [5 pt.] For the following data of the two variables X and Y; find:

i) The equation of linear regression of Y on X.

ii) The coefficient linear correlation between X and Y.

X	1	3	4	6	8	9	11	14
Y	1	2	4	4	5	7	8	9

With all best wishes
Dr.\ Waleed Elbeshbeeshy